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Evidence for recent warming from perturbed geothermal gradients: examples from eastern Canada*

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Abstract. Recent variations of the surface temperature of the Earth can be inferred from borehole temperature measurements. Generalized inversion is used to extract the information from the data; the potential of the method is evaluated. Tests were performed with synthetic data to demonstrate the effectiveness of the inversion to recover the gross features of the surface temperature history even when the data are affected by noise and errors. The tests show that it is possible to reconstruct the long term changes in ground temperature during the past 300 years; the resolution decreases with time, in particular if noise and errors must be filtered. Temperature logs, obtained in eastern Canada, and not suspected of being affected by non-climatic factors, have been inverted. The analysis confirms that eastern Canada has experienced warming by 1 to 2° C over the past 100–200 years. The relationship between air and ground temperatures has been examined. In eastern Canada ground temperature follows air temperature closely in summer but stays well above air temperature in winter. The number of days with snow on the ground correlates with the difference between annual mean ground and air temperature.

Introduction

Past variations in surface temperature of the Earth are recorded as perturbations to the steady state temperature conditions of the subsurface (Birch 1948). Subsurface temperature is commonly measured in boreholes to determine heat flow density (HFD). In eastern Canada, the deglaciation and the Holocene climatic history have caused large transient perturbations of the geothermal gradients; corrections are routinely applied to eliminate these effects and determine the steady state HFD (e.g. Birch 1948; Jessop 1971; Beck 1977). Alternatively, the transient perturbations of the geothermal gradient could be analyzed to determine past ground temperature. Temperature measurements in boreholes from many locations around the world often exhibit perturbations for the first 100 to 200 m (e.g. Cermak et al. 1984; Davis and Lewis 1984; Lachenbruch and Marshall 1986; Ballard et al. 1987; Mareschal et al. 1989). Whenever such perturbations cannot be satisfactorily explained by lateral changes in surface temperature, topography, changes in thermal conductivity, internal heat sources, etc., they are interpreted in terms of recent surface temperature variations (e.g., Cermak 1971; Beck 1982; Lachenbruch and Marshall 1986; Nielsen and Beck 1989; Beltrami and Mareschal 1991). Temperature profiles in boreholes contain information on the past ground temperature and they could thus complement the meteorological records of the recent climatic history of the continents.

The main purpose of this report is to discuss the information that can be extracted from the borehole temperature data and the quality of this information. This information is extracted by inversion of the temperature profile. Examples illustrate how the generalized inverse method (Lanczos 1961; Jackson 1972) can yield the gross surface temperature history. Tests were conducted with synthetic data, with and without noise. Past ground temperature can be determined with reasonable error bounds but relatively low resolution. Attempts to improve resolution fail and result in amplification of noise and errors to unacceptable levels. The inversion was also applied to selected temperature logs collected in eastern Canada. The results confirm the preliminary analysis of the complete data set and indicate a warming of 1 to 2° C over the last 150 years for most sites in eastern Canada (Beltrami and Mareschal 1991).

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Theoretical framework

For an homogeneous, isotropic, source-free half space, the temperature perturbation is solution of the diffusion equation in one dimension with initial and boundary conditions (Carslaw and Jaeger 1959):

$$\kappa \frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t}, \tag{1}$$

where κ is the thermal diffusivity of the rock, z is depth (positive downwards), and t is time. The use of the one-dimensional equation is valid if long term surface temperature changes have a wavelength much larger than the depth to which they penetrate (usually less than 1 km).

The present temperature perturbation $T(z)$ in a semi-infinite solid with past surface temperature $T_0(t)$, where t is time before present, is given by (e.g. Vasseur et al. 1983):

$$T(z) = \frac{z}{2\sqrt{\pi\kappa}} \int_0^\infty T_0(t)\, t^{-\frac{3}{2}} \exp\left(-\frac{z^2}{4\kappa t}\right) dt. \tag{2}$$

This expression can be integrated for various functions describing the surface temperature history.

A periodically changing surface temperature, $T_0(t) = \cos(\omega t)$ induces a perturbation $T(z,t)$:

$$T(z,t) = \cos\left(\omega t - z\sqrt{\frac{\omega}{2\kappa}}\right) \exp\left(-z\sqrt{\frac{\omega}{2\kappa}}\right). \tag{3}$$

This perturbation propagates like a wave and is damped exponentially with depth. The skin depth, at which the wave is attenuated to 1/e, is $\sqrt{\kappa/2\omega}$. The effect of periodic changes is thus negligible below a few times the skin depth. A typical value of κ for rocks is $10^{-6}\ m^2 \cdot s^{-1}$; the skin depth is on the order of 20 cm for daily variations and 3 m for the annual cycle. Between 20 and 300 m, the Earth has filtered out the high frequency part of the surface temperature signal, and recorded only the very slow variations (i.e. if the period $P = 2\pi/\omega = 10$ to 1000 years). This attenuation of the thermal perturbation with depth also reduces the amount of information extractable from the signal.

Integration of Eq. (2), for a series of N instantaneous changes of the surface temperature T_k at times t_k before present, yields:

$$T(z) = \sum_{k=1}^{N} T_k \operatorname{erfc} \frac{z}{2\sqrt{\kappa t_k}} \tag{4}$$

where erfc is the complementary error function. Figure 1 shows the temperature perturbations due to a 1 K step surface temperature increase 10, 30, 100, 300 years before present.

In the Earth, the temperature perturbation is superimposed on the equilibrium temperature. In a source-free half space, the equilibrium heat-flow is constant. The equilibrium heat flow is usually evaluated in the deeper part of the temperature profile which is not affected by the recent surface temperature changes. The equilibrium temperature is then continued upward and the perturbation is determined as the difference between the measured temperature and the upward continuation of the temperature profile. The shape of the perturbation is determined by the thermal history of the surface. This history can be inferred by comparing the calculated perturbation for a model of surface temperature with the data and adjusting the model until a fit is obtained or it can be determined directly by inversion. Different inversion techniques have been applied to this problem (e.g. Vasseur et al. 1983; Shen and Beck 1983; Nielsen and Beck 1989; Wang 1991; Shen and Beck 1991 in press). The analysis reported here is based on the generalized inverse for a linear underdetermined system of equation (Lanczos 1961).

Because short period variations can be neglected, the surface temperature can be approximated by the average surface temperature over K time intervals of equal duration Δ, i.e.:

$$T(t) = T_k \quad (k-1)\Delta \le t \le k\Delta$$

Equation (4) can then be written as:

$$\Theta_j = T_k A_{jk}, \tag{5}$$

where Θ_j is the calculated temperature perturbation at depth z_j and A_{jk} is a matrix formed by evaluating the difference between complementary error functions at depth z_j and time $t_{k-1} = (k-1)\Delta$ and $t_k = k\Delta$:

$$A_{jk} = \operatorname{erfc}\left\{\frac{z_j}{2\sqrt{\kappa t_{k-1}}}\right\} - \operatorname{erfc}\left\{\frac{z_j}{2\sqrt{\kappa t_k}}\right\} \tag{6}$$

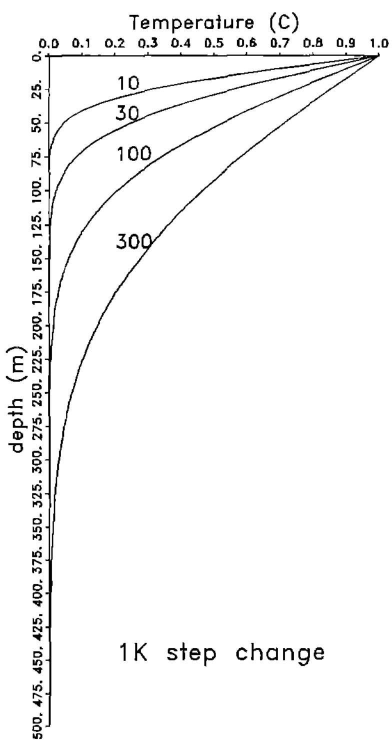

Fig. 1. Temperature perturbations for a sudden 1 K surface temperature increase starting 10, 30, 100, and 300 years before present

This yields an underdetermined system of linear equations which can be solved for T_k by singular value decomposition (SVD) (Lanczos 1961; Jackson 1972; Menke 1989). This approach is theoretically attractive because SVD effectively reduces the impact of noise and errors on the solution.

The linear transformation $\mathbf{A}\,\mathbf{x}=\mathbf{b}$ maps $\mathbf{x}$, a vector in the M dimensional model parameter space, into $\mathbf{b}$, a vector in the N dimensional data space. The singular value decomposition yields a "natural inverse" (generalized inverse) to this transformation (Lanczos 1961). It can be shown that this transformation maps K ($K \leq \min(M, N)$) orthonormal vectors $\mathbf{v}^{(k)}$ of model space into $\lambda^k\,\mathbf{u}^{(k)}$, where $\mathbf{u}^{(k)}$ are orthonormal vectors in data space, and λ^k are the singular values. M-K orthonormal vectors $\mathbf{v}'^{(k)}$ must be added to the set $\{\mathbf{v}^{(k)}\}$ to span the entire model parameter space: the transformation $\mathbf{A}$ maps these vectors $\mathbf{v}'^{(k)}$ on 0 in data space. N-K orthonormal vectors $\mathbf{u}'^{(k)}$ must be added to the set $\{\mathbf{u}^{(k)}\}$ to span the entire data space; no vector in parameter space is mapped on the subspace spanned by these vectors. The linear transformation $\mathbf{A}\,\mathbf{x}=\mathbf{b}$ can be decomposed as $\mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^{\mathrm{T}}\mathbf{x}=\mathbf{b}$ where U is the $N \times N$ orthonormal matrix formed by the vectors $\{\mathbf{u}^{(k)}\}+\{\mathbf{u}'^{(k)}\}$ spanning the data space, $\mathbf{V}$ is the $M \times M$ orthonormal matrix formed by the vectors $\{\mathbf{v}^{(k)}\}+\{\mathbf{v}'^{(k)}\}$ that span the model parameter space and $\boldsymbol{\Lambda}$ is an $N \times M$ diagonal matrix whose elements are the K singular values λ^k completed by zeros (Lanczos 1961). The generalized inverse of $\mathbf{A}$ is $\mathbf{V}\boldsymbol{\Lambda}^{-1}\mathbf{U}^{\mathrm{T}}$, where $\boldsymbol{\Lambda}^{-1}$ is the diagonal matrix formed of the elements $1/\lambda^k$ or zeros when $\lambda^k=0$. The linear system of equations $\mathbf{A}\,\mathbf{x}=\mathbf{b}$ has the solution $\mathbf{x}=\mathbf{V}\boldsymbol{\Lambda}^{-1}\mathbf{U}^{\mathrm{T}}\mathbf{b}$. The solution consists of three operations: (1) a rotation in data space ($\mathbf{U}^{\mathrm{T}}\mathbf{b}$) to determine the components of the data vector in the system of eigenvectors $\{\mathbf{u}^{(k)}\}+\{\mathbf{u}'^{(k)}\}$ (2) determination of the inverse, i.e. the vector in the subspace $\{\mathbf{v}^{(k)}\}+\{\mathbf{v}'^{(k)}\}$ that maps on the data vector ($\boldsymbol{\Lambda}^{-1}\mathbf{U}^{\mathrm{T}}\mathbf{b}$), and (3) a rotation in model parameter space to find the components of the inverse in the original model parameters $\mathbf{V}\boldsymbol{\Lambda}^{-1}\mathbf{U}^{\mathrm{T}}\mathbf{b}$. If the system is undetermined ($K<M$). the solution is not unique: any linear combination of the vectors $\{\mathbf{v}'^{(k)}\}$ can be added to the solution since it is mapped on 0 in data space. If the system is overdetermined ($K<N$), the generalized inverse is equivalent to the least square inverse (Lanczos 1961). The system can be under and overdetermined. This is the situation for inversion of surface temperature from borehole temperature measurements.

The linear combination of model parameters which have weak impact on the data correspond to the small singular values λ^k; in the inversion, the data are divided by the smallest eigenvalues. Even for noise and error free data, numerical instabilities appear when the ratio of the largest to the smallest eigenvalue exceeds a critical value (i.e. 10^{-6}); the presence of noise in the data intensifies this problem. In practice, it will thus be necessary to also eliminate from the solution all the singular values that are smaller than a fraction of the largest eigenvalue.

The impact of errors and noise on the solution is also demonstrated by the variance of the estimated parameters. The variance of the estimated parameters is given by:

$$\sigma_m^2 = \sum_{k=1}^{K} \frac{(v_{(m)}^k)^2}{(\lambda^k)^2}\,. \tag{7}$$

This variance can be understood as the amplification of the measurement errors in the solution (i.e. it is the standard error in the estimated parameter corresponding to a 1 K standard deviation in the temperature measurements). The order of magnitude of the variance is inversely proportional to the smallest singular value. For a 10^{-2} cutoff, the error on estimated parameters remains less than 100 times the measurement error. The estimated error is largest for the parameters that have a large component in the eigenvectors and smaller for those that are weakly represented in the solution. For a lower singular value cutoff, these parameters (corresponding to larger times) are more strongly represented and better resolved, but the error level becomes unacceptable (i.e. for a cutoff of 10^{-4} or 10^{-6}, the inversion amplifies 10^4 or 10^6 times the measurement errors).

The resolving power of the data and their sensitivity to noise are determined only by the properties of the eigenvalues and the eigenvectors. Figure 2 shows the eigenvectors corresponding to the eight largest eigenvalues for the inversion of temperature perturbation. For this specific example, it is assumed that the temperature perturbations are sampled every 10 m between 10 and 300 m. The model parameters are the temperatures averaged over 10 years intervals between present and 500 years ago. In principle, there are 30 non-zero singular values; however, the singular values decrease rapidly, and elimination of the smaller eigenvalues reduce the number of useful parameters to 5. This limit is not sensitive to the sampling interval and the quantity of data; in other words, the resolving power will not be improved by sampling more closely or sampling more data. Although the number of useful parameters is only 5, it is still convenient to use the higher dimensional model parameter space: the SVD will select the linear combination of the model parameters that is best resolved by the sampling of data.

Tests with synthetic examples

Synthetic data sets were created for two models of surface temperature history in the past 300 years. The first synthetic data were generated by calculating the temperature perturbation caused by two stepwise 1° C increases in surface temperature at 100 and 200 years BP. The data were sampled at 10 m interval, and the shallowest 20 m were eliminated to simulate borehole temperature measurements. The maximum sampling depth is 250 m. The effect of noise was simulated by addition of a random perturbation between −.01 and .01 K.

Figure 3a, b show the results of inversion conducted on noisefree data. In Fig. 3a, all the singular values

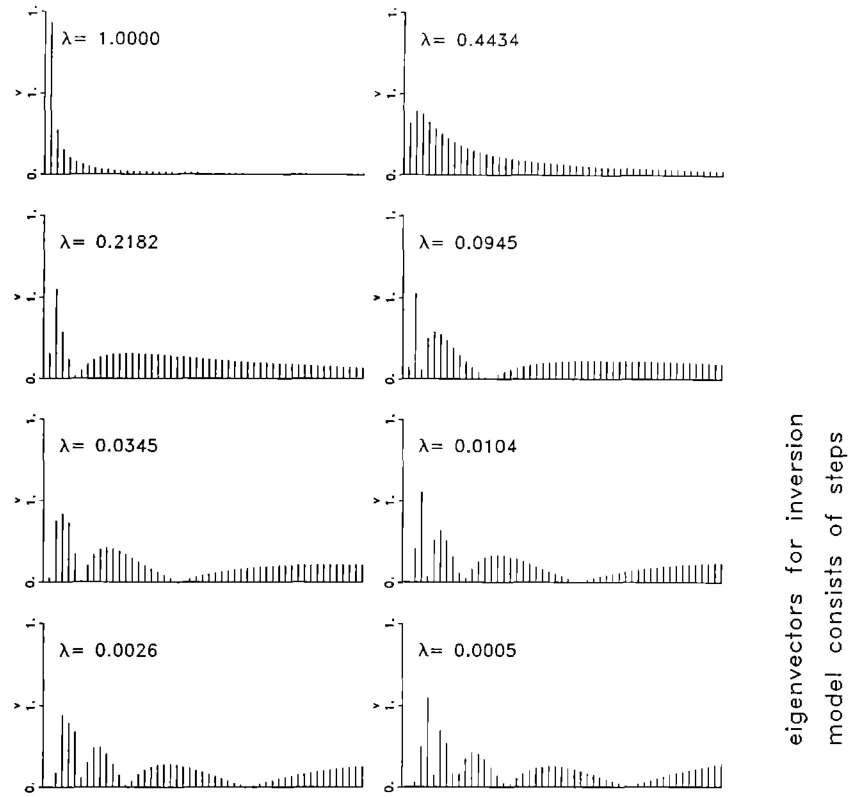

Fig. 2. Eigenvectors for inversion of surface temperature between present and 500 years ago from 25 borehole temperature samples at 10 m interval

smaller than 10^{-2} were set to zero, in Fig. 3b, the cutoff is 10^{-4}. In both cases, the surface temperature history is reasonably well approximated. It is worth noting that the lower cutoff does not improve much the result of inversion. The gross features of the solution are determined by the larger eigenvalues. Even with noise free data, some details of the temperature history are lost, regardless of the cutoff. Figure 3c, d show the results of inversion on the noisy data. The comparison illustrates perfectly the impact of the noise when the smaller eigenvalues are retained. The impact of noise on the solution is minimal for a 10^{-2} cutoff, but it is dramatic if the cutoff is lowered to 10^{-4}. Examination of the variance of the parameters leads to the same conclusion (i.e. if the standard deviation on temperature measurements is .01 K, the standard error overwhelms the signal if the singular value cutoff is less than 10^{-2}). These examples demonstrate that elimination of the smaller eigenvalues reduces the impact of noise without altering the gross features of the solution.

A second set of synthetic data was generated to study whether past oscillations in surface temperature can be detected. The temperature perturbation caused by a 1° C drop in surface temperature between 250 and 100 years BP was calculated. Noise was added as in the previous example. Figure 4a, b show the reconstructed temperature history for different eigenvalue cutoffs. The temperature oscillation is better detected with a low (10^{-4}) cutoff.

Because the system of equations is underdetermined, the solution obtained is not unique. All the other solutions can be generated by adding to the special solution any linear combination of the eigenvectors (in model parameter space) with singular values equal to zero. These eigenvectors correspond to rapid oscillations of the surface temperature (the period of these oscillations increases from 20 years for the most recent to 100 years for the most ancient). In other words, the singular value decompositon will naturally eliminate from the inverse all the oscillations of the surface temperature that the data can not resolve and it will yield the average surface temperature. A crude estimate of the resolving power of the data can be obtained by calculating the maximum temperature perturbation due to a change of T degrees, during a time interval Δt, starting t years ago. It is given by: $T_{max}=0.24\,T\Delta\,t/t$. If the detection threshold is 0.05 K, $T\Delta t/t=0.21$. In other words, a 1° C perturbation can be detected only if its duration is at least 20% of the time of occurence.

Interpretation of data from eastern Canada

For this study, three temperature logs collected for HFD determination in eastern Canada (Mareschal et al. 1989; Pinet et al. 1991), were analyzed to extract recent surface temperature. These sites are located in Belleterre (47 24° N 78 42° W), Val d'Or (48 05° N 77 33° W), and Evain (48 17° N 79 06° W). These mining exploration boreholes were drilled 1985, 1987, and 1980 respectively, and they were logged for temperature in 1987 and 1988. They are situated in forested, relatively flat, and undeveloped land. The sites are thus not affected by differences in solar exposition. These sites are also isolated from cultural activities (i.e. roads, cities, farming, etc.) and are not affected by landscape disrup-

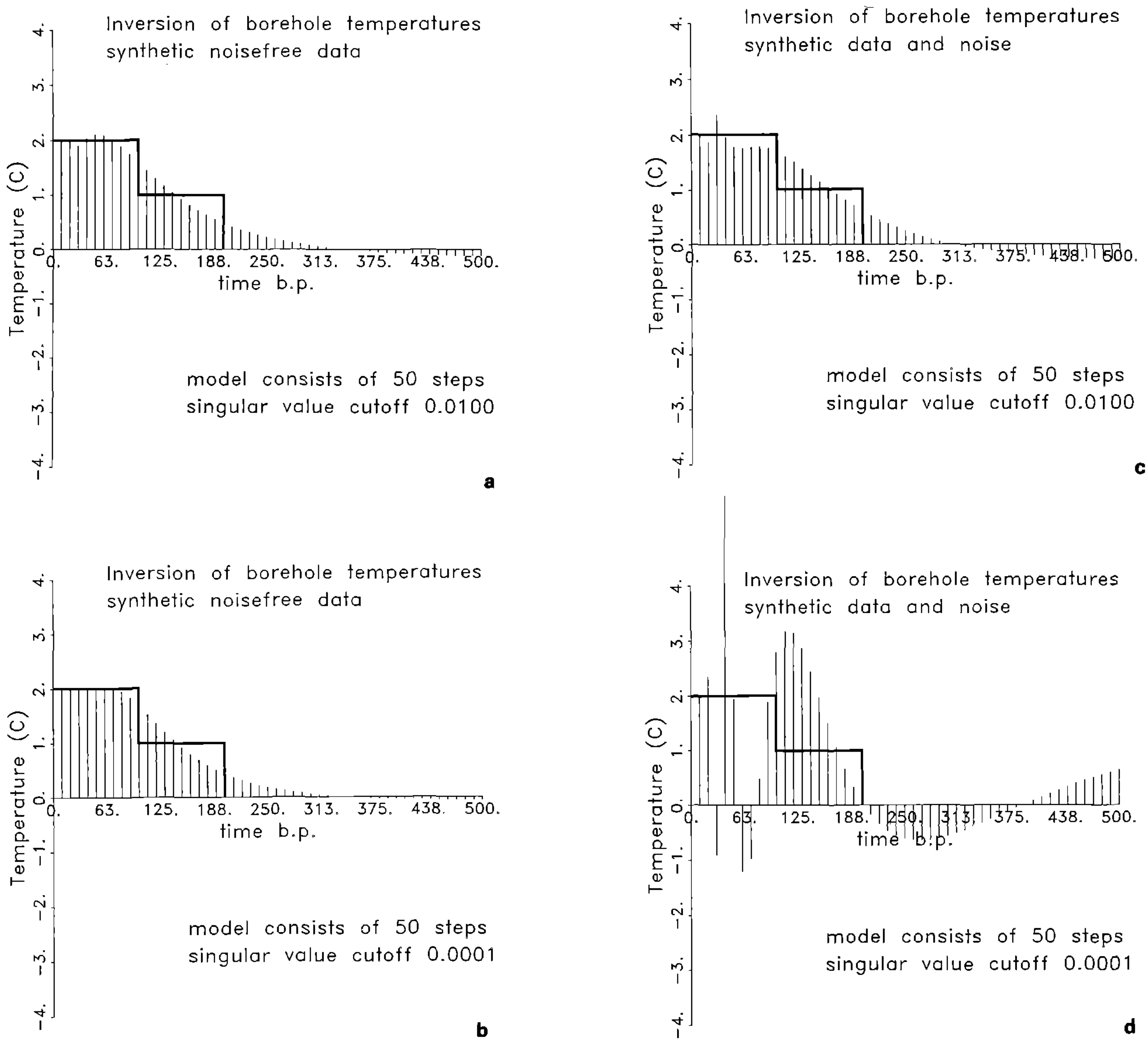

Fig. 3a-d. Inversion of synthetic borehole temperature data. The *solid line* represent the surface temperature history used to generate the data (a 1 K sudden increase 200 years ago, and a 1 K increase 100 years ago). *Vertical lines* are the surface temperature history obtained by inversion. **a** Noisefree data with cutoff at 10^{-2}; **b** Noisefree data with cutoff at 10^{-4}; **c** Noisy data with cutoff at 10^{-2}; **d** Noisy data with cutoff at 10^{-4}

tion. Temperature measurements were made at 10 m intervals with a calibrated thermistor. The shallowest part of the log is missing because measurements were made only below the water table, and the data above 20 m are discarded to avoid the effect of seasonal temperature variations.

For each temperature log, the "undisturbed" heat flow is determined following standard methods in the deepest part of the borehole (Bullard 1939; Powell et al. 1988). The "steady state" temperature is then continued upward to the surface by assuming constant gradient over intervals of constant conductivity. The temperature perturbation is the difference between the observed temperature and the upward continued steady-state temperature profile. The upward continued temperature to the surface gives the mean ground temperature before the onset of the perturbation. The extrapolation of the observed profile approximates the surface temperature at about one decade before measurements.

Figure 5 shows the profiles selected for inversion, and the upward continued temperature. The result of inversion for the sites at Beleterre, Val d'Or, and Evain are shown in Fig. 6a, b, and c respectively. The details of surface temperature history at each site vary, but some consistent trends appear: surface temperature has recently increased by 1.5 to 2.0° C; the warming started 120 years BP at Val d'Or and Belleterre, and sooner (ca. 300 years BP) at Evain. The difference between the results at Evain and the other sites deserves further investigation. The borehole at Evain is deeper (600 m versus 300-400 m), and it is possible that the data, at Val d'Or and Belleterre, are not sufficient to detect any surface temperature change before 150 years BP. Deeper holes in the Val d'Or area will be logged and analyzed. All three surface temperature histories exhibit a short inter-

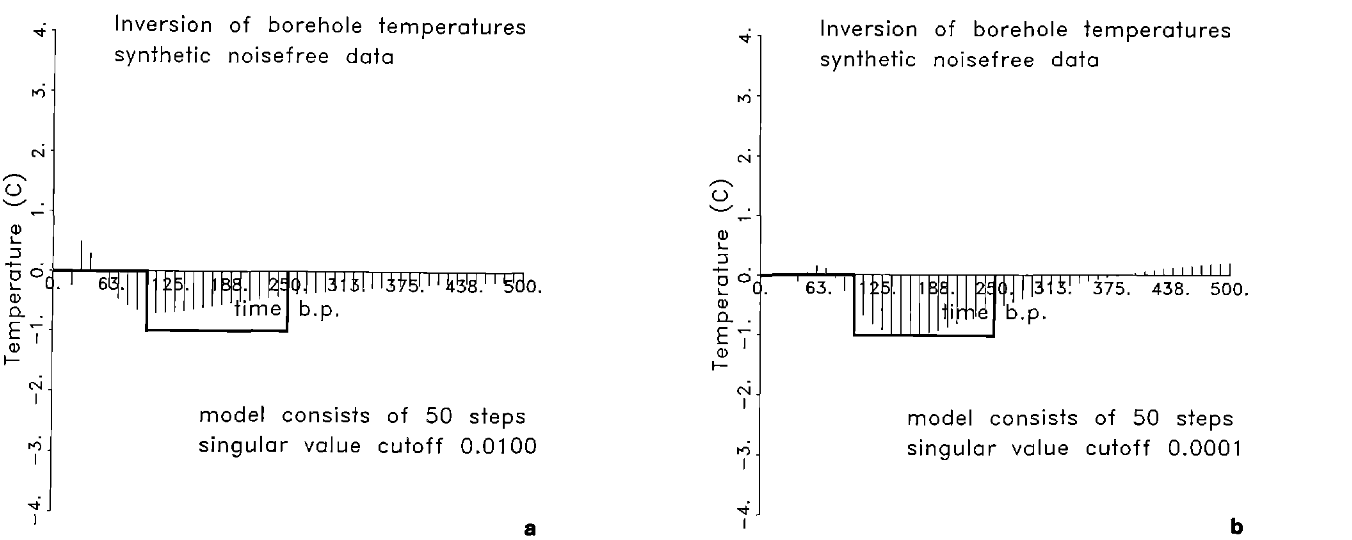

Fig. 4a, b. Inversion of synthetic borehole temperature data. The *solid line* represent the surface temperature history used to generate the data (a 1 K sudden decrease 250 years ago, and a 1 K increase 100 years ago. **a** Noisefree data with cutoff at 10^{-2}; **b** Noisefree data with cutoff at 10^{-4}

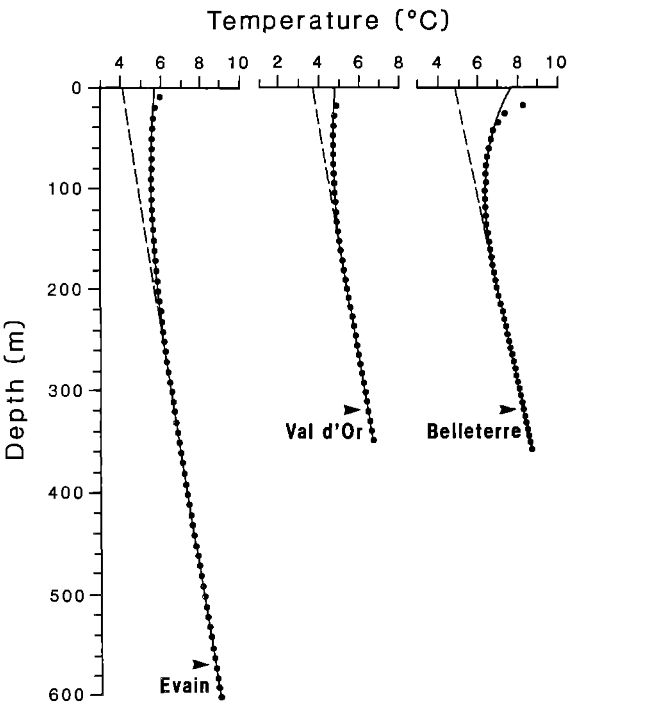

Fig. 5. Temperature profiles at Val d'Or, Evain, and Belleterre. The *dots* represent the measured temperature. The *line* is the upward continued temperature profile

val with colder temperature 20 to 30 years ago. This short interval seems a robust conclusion that does not depend on the inversion scheme; it was also reached from independent analysis of the same data with a different method (K. Wang, personal communication).

Discussion and conclusions

The detailed analysis of borehole temperature data in eastern Canada confirms the conclusion of several similar studies. Nielsen and Beck (1989) have analyzed four boreholes in Ontario and also concluded in recent warming. A preliminary study by Jessop (personal communication) indicates warming in most of Canada, east of Hudson Bay. Beltrami and Mareschal (1991) analyzed all the boreholes available to them and reported warming at 12 sites in Ontario and Quebec; 6 other sites yielded insufficient or contradictory evidence (but no sign of cooling). The results of this analysis is presented on Fig. 7. The warming reported in eastern Canada is consistent with the analysis of meteorological trends by Hansen and Lebedeff (1987). It is also consistent with the analysis of local meteorological data (Diaz and Bradley, Presentation at Clima Locarno 90 Conference).

The analysis raises two fundamental questions: (1) whether the surface temperature history extracted from geothermal measurements reflects climatic variations, and (2) how is the ground temperature related to air temperature and to the precipitation regime.

Horizontal variations of surface temperature can distort the temperature profile. The temperature perturbation for a surface temperature of the form $T_0 \cos(kx)$ is:

$$T(x,z) = T_0 \cos(k,x) \exp(-kz) \tag{8}$$

and the maximum vertical gradient at $x=0$ is:

$$\frac{\partial T}{\partial z} = -k T_0 \exp(-kz). \tag{9}$$

For a 1° C temperature difference over 100 m (i.e. the half wavelength = $\pi/k = 100$ m), the perturbation of the vertical gradient is about 30 K/km; it is larger

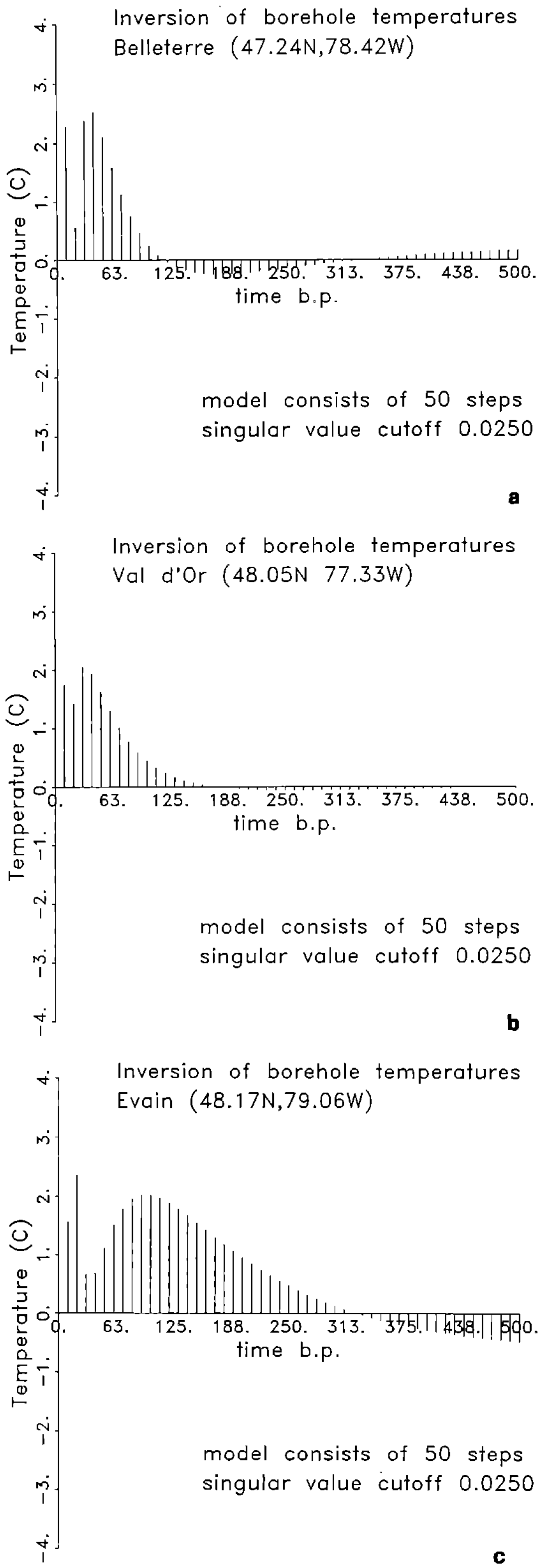

Fig. 6a–c. Temperature history from inversion of the borehole data. **a** Belleterre (QC); **b** Val d'Or (QC); **c** Evain (QC)

than some of the perturbations observed in eastern Canada. It is attenuated exponentially with depth with a "skin depth" on the order of 1/k (30 m for the example above). Differences of surface temperatures of 1 to 2° C are commonly observed between forested and cleared adjacent areas (Powell et al. 1988). In the authors' opinion, horizontal surface temperature variations cannot explain the reduced or negative gradients observed in eastern Canada. If sampling is not biased, gradients should be decreased as often as increased. Almost all the boreholes are located in forested areas, but are not close to clearings and should thus not be affected by horizontal variations in surface temperature. This remains a potential problem and requires careful selection of the boreholes before analysis.

In eastern Canada the difference between air and ground temperatures varies from 1.5 to 6° C depending on latitude (Beltrami and Mareschal 1991). This difference is mainly due to the insulation of the ground by snow during winter months. This effect is illustrated in Fig. 8a, which shows measured monthly mean air and soil (5 cm) temperatures at Val d'Or (QC) (data from Environment Canada). During the winter months, snow insulates the ground and keeps ground temperature near freezing, while mean air temperature drops to −15° C to −20° C. During the period analyzed, the mean annual air and soil temperatures were 1.3 and 6.5° C respectively. In eastern Canada, the number of days with snow on the ground is correlated with air/soil temperature differences. Figure 8b shows the difference between mean annual air and soil (3 m) temperature and the number of days with snow on the ground during April, October, and November. The number of days with snow on the ground appears as the main factor accounting for air-ground temperature differences in eastern Canada. As a result, long-term changes in snow cover might interfere with the detection of a trend in air temperature from subsurface measurements.

To improve the resolution of surface temperature history, complementary information can be obtained by measuring the rate of change of the temperature perturbation (Lachenbruch and Marshall 1986; Nielsen and Beck 1989). The time derivative of the temperature for a stepwise change in surface temperature is:

$$\frac{\delta T(z)}{\delta t} = T_0 \frac{z}{2\sqrt{\pi \kappa t^3}} \exp\left\{-\frac{z}{4\kappa t}\right\}, \qquad (10)$$

which is maximum at depth $z=\sqrt{2\kappa t}$. For a 1° C increase 100 y BP the depth of the maximum is 75 m and the temperature change in 10 years will be 0.024 K. For the same change 50 and 20 y BP, the maximum rate of change will be at 56 and 36 m, with changes of 0.05 and 0.12 K over 10 years. For the warming found in eastern Canada, the maximum change in temperature should be about 50 m K over 10 years. These changes should be easily detected with existing technology. The data could improve the resolution of the inversion scheme. This information can also be used to differentiate between stationary and transient perturbations to the geothermal gradient.

The difficulty of extracting information on recent climatic changes from meteorological records is well known (Karl et al. 1989). Temperature measurements in boreholes can be used to complement the meteorological record. Because the Earth has indeed recorded its

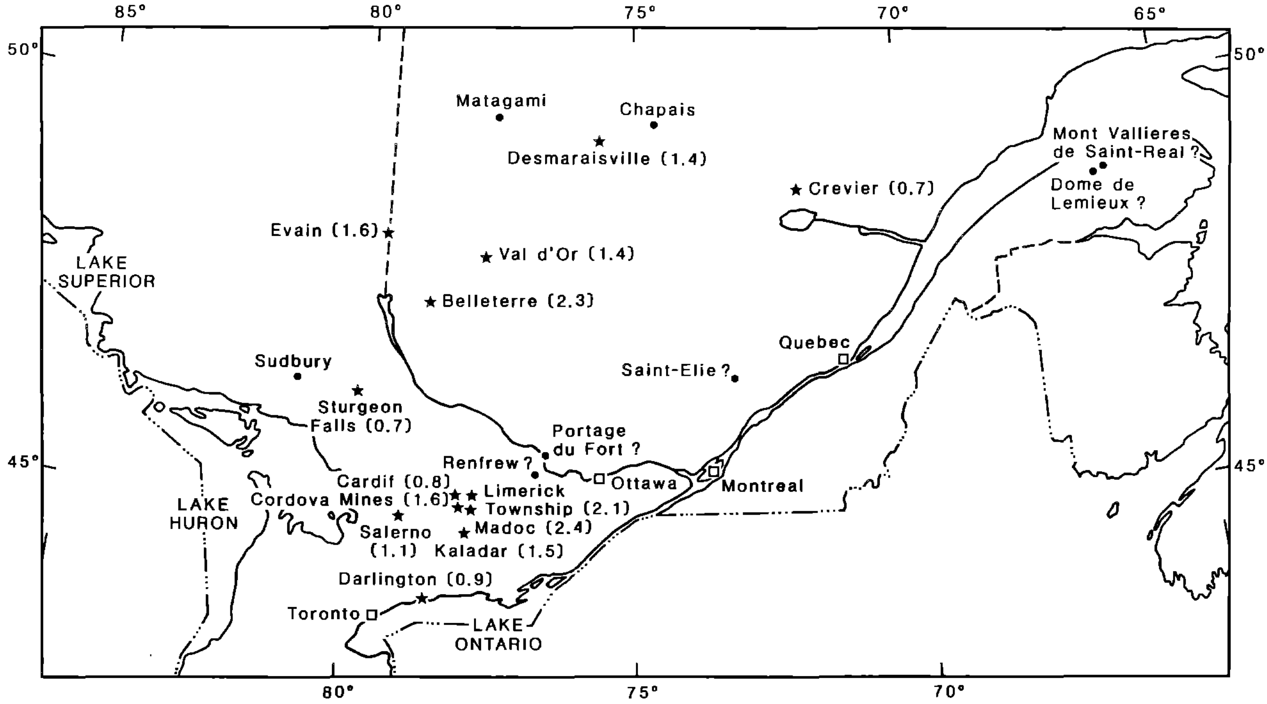

Fig. 7. Preliminary interpretation of the sites measured in eastern Canada. *Stars* mark sites with clear signs of warming (the inferred temperature change is given in brackets). *Dots* represent sites with no clear indication of warming. Perturbations at Matagami, Chapais, and Sudbury could not be interpreted because of noise (groundwater flow, effect of a lake)

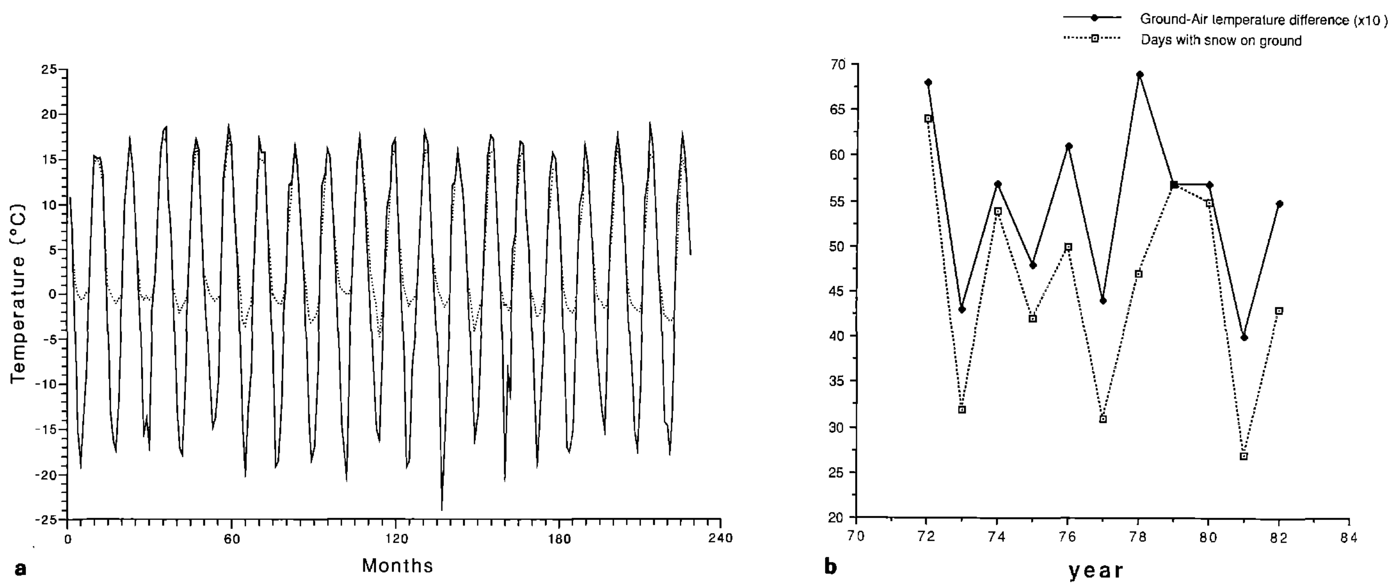

Fig. 8a, b. Relationship between soil temperature and snow cover. **a** Monthly mean air temperature and soil temperature (5 cm) at Val d'Or (QC) for the period between 1972 and 1989. The mean annual air/soil temperature difference for this location is 5.2° C; **b** Air-soil temperature difference and number of days with snow on the ground (April, October and November) as a function of time for Val d'Or

past surface temperature everywhere, dedicated boreholes could be drilled to obtain information where meteorological records are deficient. Temperature measurements in boreholes offer the advantage that the Earth has filtered out short-period and recorded only long-term trends in surface temperature variations. Careful inversion of the data can be used to reconstruct these changes.

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Note added in proof. Inversion of temperature data from deeper boreholes show a marked temperature minimum 200-300 years ago during the peak of the Little Ice Ages and confirm a trend that appears in Figure 6a.