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Surface Heat Flux Histories from Geothermal Data: Inferences from Inversion

Hugo Beltrami

Department of Geology, St. Francis Xavier University, Antigonish, Nova Scotia Canada

Abstract. Past changes in the Earth's surface energy balance propagate into the subsurface and appear as perturbations of the subsurface thermal regime. A singular value decomposition inversion (SVD) scheme has been used to reconstruct surface heat flux histories (SHFH) from the heat flux anomalies detected in the subsurface. Synthetic data tests yield results in agreement with analytical expressions. Inversion of a temperature log from central Canada is used to illustrate this procedure. Results show that reconstruction of SHFH is possible with existing data. Application of this method to the existing geothermal data base should permit a quantification of the energy balance at the Earth's surface for the past few centuries.

Introduction

To date there has been no work to directly infer variations of surface heat fluxes from geothermal data. Determination of the Earth's long-term surface heat flux is an important problem in the study of climate change because energy balance variation at the land-atmosphere interface is a fundamental quantity for general circulation models (GCMs) [*Koster and Suarez*, 1992; *Gupta et al.*, 1999]. The total contribution of radiative forcing associated with greenhouse gas emissions from anthropogenic activities to the energy balance at the Earth's surface is estimated to be about 2.0-2.5 W m^{-2} since 1765 [*Houghton et al.*, 1990]. About one third of this forcing is direct radiative heating of the surface, and about 10 % of this flows into the ground [*Sellers*, 1995] This forcing is spatially and temporally variable such that regional variations are expected. Such a small yet important component in the energy balance of the earth's surface and its spatial and temporal variability are difficult to measure accurately from meteorological data due to uncertainties in the measurements of atmospheric variables [Karl et al., 1989] and also because of the complex and complicated processes at the air-ground interface [*Geiger*, 1965]. Geothermal data, on the other hand, contain useful information about the signatures of long term climatic scenarios because the Earth behaves as a low pass filter retaining the long-term trends of climate recorded as variations of ground temperature. For example, daily and annual temperature variations are detectable to depths of 1 m and 20 m respectively; a temperature change of 1°K in the course of a 100-year period is detectable 100 m into the subsurface [*Lettau*, 1951]. Several approaches have been developed to reconstruct ground surface temperature histories (GSTHs) from borehole temperature profiles [*Lachenbruch and Marshall*, 1986; *Lewis*, 1992]. It has been learned that small changes in the energy balance at the earth's surface will be reflected in geothermal records as long as the underlying physical processes are sustained. In fact, the magnitudes of the ground surface heat fluxes estimated from temperature-depth anomalies at some locations are on the order of 2 to 5 mW m^{-2} [*Putnam and Chapman*, 1996] averaged over a period of 100 years. It is important that fluxes be estimated over large scales with time resolution sufficient to resolve the temporal variability of the energy balance at the Earth's surface.

Although there exist considerable work on the determination of GSTHs from geothermal data [*Pollack and Huang*, 2000], the extraction of temporal and spatial surface heat flux variations from these data has remained unexplored. Beltrami et al. (2000) presented the first attempt to estimate fluxes from existing GSTHs using an integral approximation. In this paper I present an application of SVD inversion to retrieve the heat flux history directly from temperature-depth profiles. It is found that the inversion procedure recovers the surface heat flux histories (SHFH) when tested with synthetic data and that it is consistent with heat-flux history estimates reconstructed by a discrete approximation derived analytically. The methodology is illustrated with an analysis of a temperature-depth profile measured in central Canada. Application of this method to reconstruct SHFH using larger data sets at regional and global scales [Huang et al., 2000] should be straight-forward.

Theoretical Framework

In an homogeneous semi-infinite, source-free half space the temperature T at depth z due to a time varying surface temperature change is governed by the one-dimensional unsteady heat diffusion equation with boundary and initial conditions[*Carslaw and Jaeger*, 1959]:

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}, \tag{1}$$

where t is time and κ is the thermal diffusivity. Because the heat flux is related to the temperature gradient by Fourier's equation in one dimension,

$$q = -\lambda \frac{\partial T}{\partial z}, \tag{2}$$

where λ is the thermal conductivity. Substituting Eq. 2 into Eq. 1 yields,

$$\frac{\partial q}{\partial t} = \kappa \frac{\partial^2 q}{\partial z^2}. \tag{3}$$

So, heat flux variations within the semi-infinite half space satisfy the same diffusion equation as temperature variations

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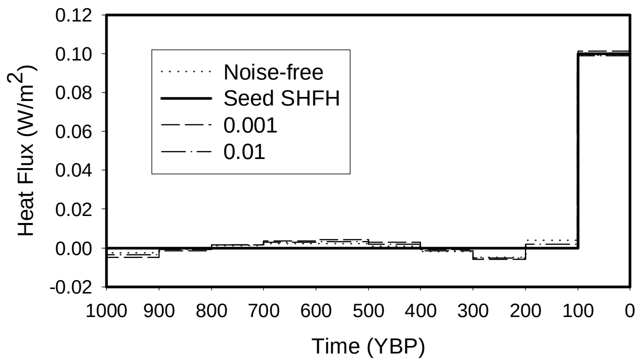

Figure 1. SHFHs for indicated noise levels (Wm^{-2}) of single step synthetic test.

[*Carslaw and Jaeger*, 1959; *Turcotte and Schubert*, 1982]. If the semi-infinite space is subjected to a constant heat flux, q_0, at the surface under the assumptions that initially the temperature of the half space is T_0 and,

$$q = q_0, \quad \text{for} \quad t = 0, \quad z > 0, \tag{4}$$

$$q = q_0, \quad \text{for} \quad t > 0, \quad z \to \infty \tag{5}$$

Hence, the heat flux anomaly at depth z is given by

$$q(z,t) = q_0 erfc(\frac{z}{2\sqrt{\kappa t}}). \tag{6}$$

In the Earth, the heat flux at any depth can be expressed as the contributions of the quasi steady-state geothermal heat flux, q_{eq}, and the transient component $\Delta q_0(z,t)$, that is:

$$q(z,t) = q_{eq} + \Delta q_0(z,t). \tag{7}$$

Geothermal data are usually acquired as temperature-depth profiles such that it is possible to calculate the heat flux variation as a function of depth using Eq. 2. The steady state geothermal heat flux can be determined from the deepest part of the temperature profile which is least affected by recent surface temperature variations. The SHFH can be approximated by a series of step heat flux changes at the surface [*Beltrami et al.*, 1992], such that subtracting the estimated q_{eq}, the heat flux anomaly at depth z, is given by:

$$\Delta q_t(z) = \sum_{k=1}^{K} q_k \left(erfc(\frac{z}{2\sqrt{\kappa t_k}}) - erfc(\frac{z}{2\sqrt{\kappa t_{k-1}}}) \right) \tag{8}$$

$$q_0(t) = q_k; \quad t_{k-1} \le t \le t_k \; k = 1, ..., K; \; t_0 = 0. \tag{9}$$

Equation 8 is evaluated at each depth where data exist, forming a system of linear equations on k unknowns which can be inverted by SVD to yield a series of surface heat flux values representing the SHFH at the site.

Analysis

As an initial test, a synthetic SHFH consisting of a one step change of surface heat flux of $0.1 Wm^{-2}$ taking place 100 ybp was generated. The forward problem described by Eq. 6 was used to generate a q-depth perturbation profile. Inversion of this profile for a single step (Lachenbruch and Marshall, 1986) recovered surface heat flux magnitudes of 0.09966, 0.09969 and 0.09996 Wm^{-2} for added white noise levels of ± 0.01, ± 0.005 and ± 0.001 Wm^{-2} respectively. The corresponding temperature depth anomaly profile for this SHFH can be generated [*Turcotte and Schubert*, 1982] assuming a constant thermal conductivity ($\lambda = 3.0 W/mK$) from:

$$T(z,t) = \frac{2q_0}{\lambda}\sqrt{\frac{\kappa t}{\pi}}, \tag{10}$$

where $\kappa = 10^{-6} m^2 s^{-1}$. SVD inversion of this temperature anomaly yields [*Beltrami et al.*, 1997] a one step surface temperature change with $\Delta T = 1.80°K$. The average surface heat flux due to a single step temperature change can be approximated by [Lachenbruch et al., 1982]

$$q_{ave} = \frac{2\lambda\Delta T}{\sqrt{\pi\kappa t}}, \tag{11}$$

which in this case yields 0.108 W/m^2 as average surface heat flux. This differs from the expected result by only 8%. Inversion results from the flux-depth anomaly profile for a series of 10 equal steps model parameters for different noise levels are shown in Fig. 1. This figure illustrates that recovery of the seed SHFH is also possible for a less constrained model with noisy data.

A synthetic GSTH formed by several step temperature changes (Fig. 2a, thick line) was also used as a forward model to generate a temperature perturbation profile from which the flux perturbation profile was determined. Inversion of the flux anomaly yields the SHFH shown in Fig. 2b (solid). Verification of this result can be accomplished by inverting the temperature anomaly profile to recover the synthetic GSTH. This is possible provided the same parametrization is used to invert for GSTH and SHFH. The GSTH recovered (Fig.2a, grey) is typical of this type of inversion. It shows loss of resolution with increasing time, and the inversion, as expected, does not recover the "sharp

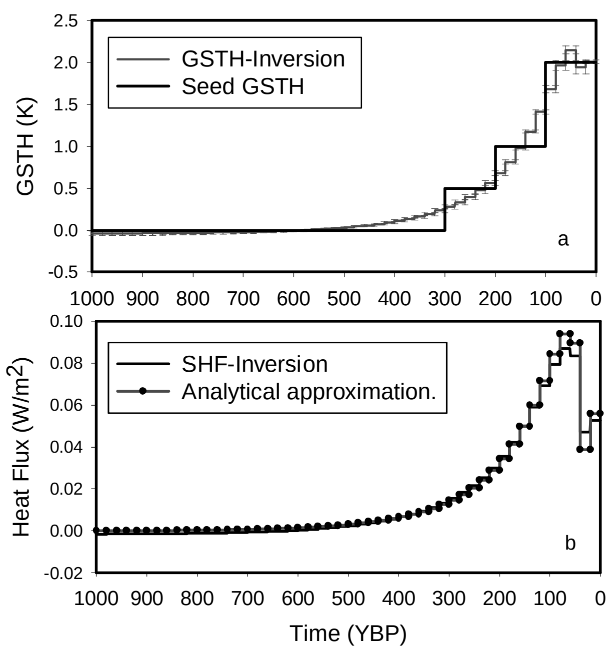

Figure 2. a. GSTH recovered by inversion from synthetic temperature profile (grey). Seed GSTH (dark). **b**. SHFH from inversion of flux-depth anomaly generated from GSTH in **a** (dark), and SHFH from analytical approximation (dots).

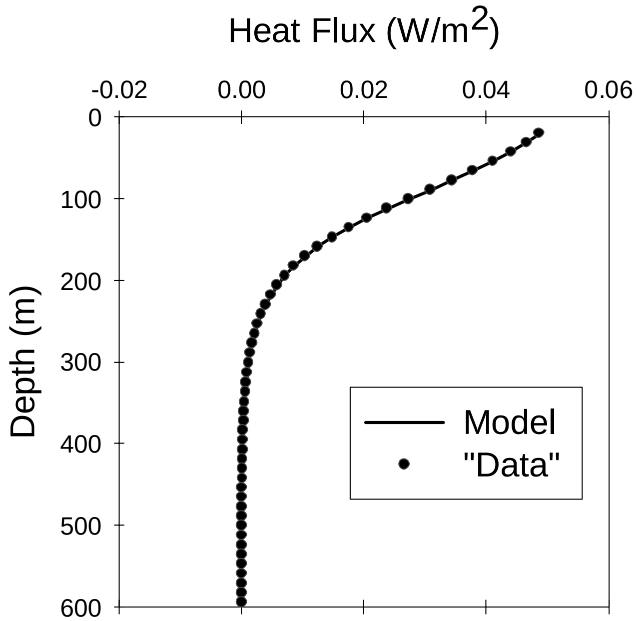

Figure 3. Heat flux anomaly and fitted profiles for test shown in Fig. 2. See text.

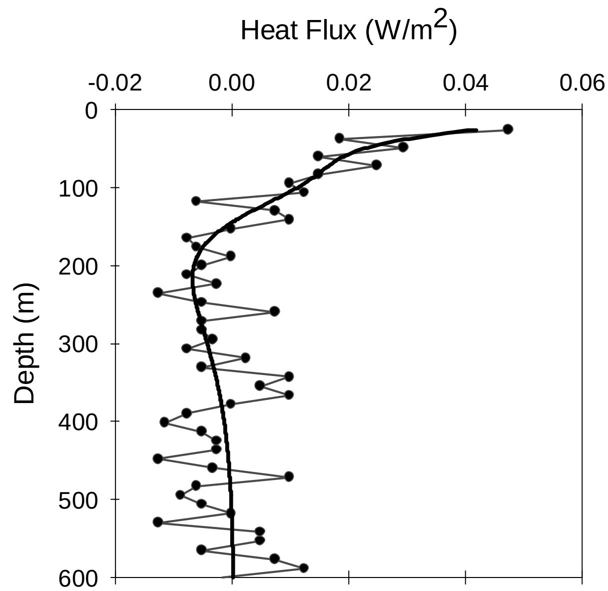

Figure 5. Heat flux anomaly and fitted profile from the inversion shown in Fig. 4. Sampling rate is approximately 1m. Fewer data are plotted for clarity.

edges" [*Beltrami and Mareschal*, 1995]. Is this GSTH consistent with the SHFH obtained above? This can be answered by reconstructing the SHFH using an analytical approximation to obtain average heat fluxes at the surface from the GSTH itself (Beltrami et al., 2000). Figure 2b show the SHFH obtained from inversion and the one obtained using the analytical approximation. The discrepancies between analytical and inverted SHFHs reconstructions in the most recent past are due to error accumulation from the analytical approximation method. That is, the error function treatment diverges at $t = 0$ such that an average integral is used [Lachenbruch et al., 1982]. The forward problem generated from the solution, and the flux-depth anomaly profile are shown in Fig. 3, where is evident that that the model can explain this flux-depth anomaly. Evidently, this is a self consistent situation and one can be confident that SHFHs can be recovered from inversion of flux anomaly profiles.

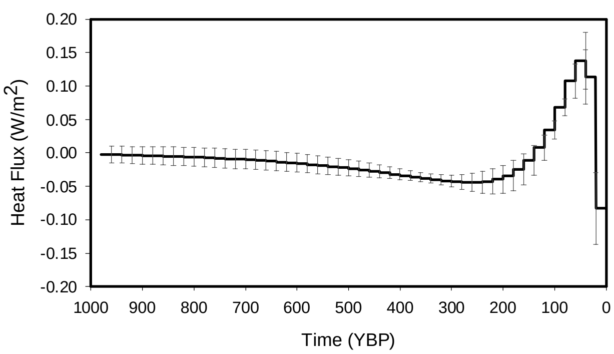

Figure 4. SHFH for Minchin Lake obtained from inversion. Error bars indicate the standard error of the estimated parameters for a noise level of $\pm 0.01 W/m^2$

A Real Data Example

A temperature-depth profile and thermal conductivity data for Minchin Lake in Ontario, Canada [*Neilsen and Beck*, 1989] was chosen to illustrate the application of the method. The temperature perturbation was first evaluated in the standard manner [*Lachenbruch and Marshall*, 1986] as the difference between the measured temperature and the steady state profile extrapolated from the deepest parts of the profile to the surface. A flux anomaly profile was generated using Eq. 2 and the average value of the thermal conductivity ($\lambda = 3.0\ W/m.K$) for this site. Assuming uniform λ is reasonable because there is little departure from the mean for the 74 thermal conductivity measurements available for this site. The equilibrium geothermal heat flow density ($q_{eq} = 34.5\ mWm^{-2}$) was subtracted from the heat flux profile to yield a flux anomaly profile. Inversion was carried out for a model consisting of fifty 20-year step surface heat flux changes for an eigenvalue cutoff of 0.04 (i.e. 5 eigenvalues retained for the inversion). The resulting SHFH for the site is shown in Fig. 4, along with the standard errors of the estimated parameters for a noise level $\pm 0.01\ Wm^{-2}$. Figure 5 shows the heat flux anomaly profile obtained at this site together with the profile generated from the inversion solution.

Discussion and Conclusions

It has been shown that for synthetic data tests, an inversion procedure based on SVD, recovers the expected SHFHs. These results are also in good agreement with the analytical approximation for these cases. Application of this inversion to a temperature depth profile in Canada yielded an average heat flux over the last 1000 years on the order of -9.44 mWm^{-2}. The heat flux since 1765 is 19.5 mWm^{-2} and 69.0 mWm^{-2} for the last 100 years. Note that according to Fig. 4, the largest negative heat flux into the ground occurred 250-300 years before present. GSTH from geothermal data across large regions of Canada have shown a minimum ground surface temperature 150 to 200 ybp [*Lewis*, 1992]. Whether this discrepancy is present in other regions of Canada requires the analysis of the complete Canadian dataset.

The method illustrated here treats the subsurface as a homogeneous medium. Although this is not the case at all locations, most geothermal data are accompanied by limited thermal conductivity data and so, the homogeneous approximation should not be too restrictive. However, extension of this methodology to include thermal conductivity variations, simultaneous inversion for the steady state geothermal heat flux and SHFH can be readily established.

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Hugo Beltrami, Department of Geology, St. Francis Xavier University, P.O. Box 5000, Antigonish, Nova Scotia, Canada, B2G 2W5, email: hugo@stfx.ca

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